ERL Gradient and Phase Tolerance Calculations

2007 Energy Recovery Linac Workshop
Daresbury Laboratory, UK: SRF-WG3

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May 23, 2007

Outline

- Motivation
- Advanced Photon Source (APS) ERL concept
- Calculations
  - First pass (user) beam energy error and spread
  - First pass beam gradient and phase tolerances
  - Energy recovered (second pass) beam energy error and spread
  - Energy recovered beam effective energy spread based on first pass gradient and phase error tolerances
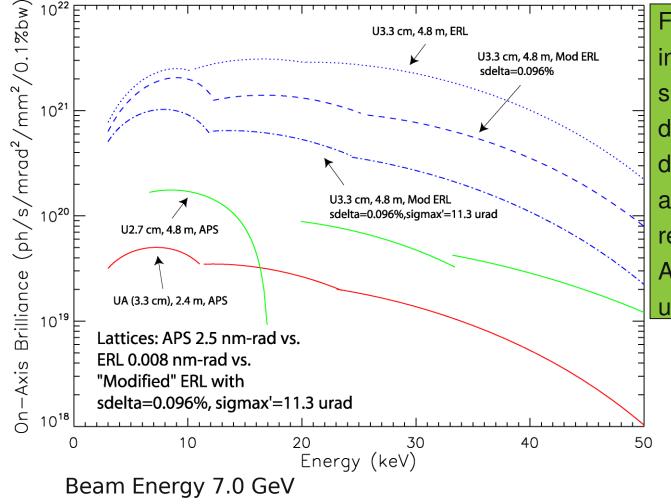- Conclusion

Motivation

- APS has users that need X-rays up to as high as 100 keV
- Energy spread impact on brightness of undulator X-rays in higher harmonics above 10 keV is large
- ERL has potentially a factor of 5 smaller energy spread (0.02%) than the APS storage ring (0.096%) @ 7 GeV
- However, random energy jitter due to gradient and phase jitter in the cavities increases the effective energy spread
  - Gradient and phase feedback loops have a finite bandwidth
  - Left over cavity gradient and phase fluctuations will add in quadrature to the natural energy spread
- **How much is the effective beam energy spread increased due to uncorrectable gradient and phase fluctuations?**
  - **User beam**
  - **Energy recovered beam**

On-Axis Brilliance Tuning Curves for APS ERL with Artificially Changed Beam Energy Spread and Horizontal Divergence[1]

For the ERL, increased beam energy spread and horizontal divergence each decreases brilliance ~3x at high energies and recovers the shape of the APS 3.3-cm-period undulator tuning curve.

Beam Energy 7.0 GeV
Beam Current 100 mA (APS), 25 mA (ERL)
Beam Energy Spread 0.096% (APS), 0.020% (ERL)

[1]Courtesy Roger Dejus APS.

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Ultimate APS ERL Upgrade Concept[1]

- Single-pass 7 GeV linac points away from APS to permit straight-ahead hard x-ray short-pulse facility[2,3]

- Beam goes first into new, emittance-preserving turn-around/user arc[4]
  - Second-stage upgrade would add many new beamlines

- ERL can benefit from very long undulators[5]
  - Higher flux and brightness
  - Could add these using somewhat different geometry

- Ability to store beam unchanged[1]

- Existing injector complex unchanged.

[1] G. Decker, OAG-TN-2006-058, 9/30/06.

[2] M. Borland, "ERL Upgrade Options and Possible Performance," 9/18/06.

[3] M. Borland, "Can APS Compete with the Next Generation?", May 2002.

[4] M. Borland, OAG-TN-2006-031, 8/16/06.

[5] S. Gruner et al., "Synchrotron Radiation Sources for the Future," 11/30/200.

[1] Slide courtesy of M. Borland APS.

Outline of Calculations

- Define the "energy error" for first pass (user) and energy recovered beams due to random gradient and phase errors
- Define a relative energy error at a specific beam energy in terms of the energy error
- Assume gaussian (normal) error distributions for gradient and phase errors
- Derive an analytic formulas for the relative energy error
- Compare the accuracy of these formulas to a "monte-carlo" calculation of the relative energy error
- Define the effective energy spread in terms of the natural energy spread and relative energy error
- Use the analytic formula to evaluate gradient and phase error impact on the effective energy spread of user and energy recovered beams

User (First Pass) Beam Analytic Formula Derivation

- User (first pass) beam energy error for an *N* cavity linac:

$$\Delta E = E_{tot} - E_f = \sum_{n=1}^{N} (V_o + \Delta V_n)\cos(\phi_o + \Delta\phi_n) - NV_o\cos\phi_o$$

$$E_f = E_o + NV_o\cos\phi_o$$

- Definition of the "relative" energy error (squared):

$$\sigma^2_{\left(\frac{\Delta E}{E_f}\right)} \equiv \frac{\langle(\Delta E - \langle\Delta E\rangle)^2\rangle}{E_f^2} = \frac{\langle\Delta E^2\rangle - \langle\Delta E\rangle^2}{E_f^2}$$

- Evaluate sums over *N* cavities using the approximation:

$$\langle f(x)\rangle \approx \frac{1}{N}\sum_{n=1}^{N} f(x_n) \approx \frac{1}{\sqrt{2\pi}\sigma_x}\int_{-\infty}^{\infty} e^{-x^2/(2\sigma_x^2)} f(x)\,dx$$

User (First Pass) Beam Analytic Formula Derivation cont.

- Relative energy error for the first pass beam:
  - $\sigma_{\Delta V}$ is the absolute gradient error
  - $\sigma_{\Delta \phi}$ is the absolute phase error

$$\sigma^2_{\left(\frac{\Delta E}{E_f}\right)} = \frac{1+\cos 2\phi_o \, e^{-2\sigma^2_{\Delta\phi}}}{2N\cos^2\phi_o}\left(\frac{\sigma_{\Delta V}}{V_o}\right)^2 + \frac{(1-e^{-\sigma^2_{\Delta\phi}})(1-\cos 2\phi_o \, e^{-\sigma^2_{\Delta\phi}})}{2N\cos^2\phi_o}$$

- Effect of gradient and phase errors decreases for long linacs
- Phase errors much less important than gradient errors for on-crest phasing ($\phi_o = 0$)
- How accurate is the formula compared to direct calculation of the relative energy error?
  - Calculate via monte-carlo the relative energy error
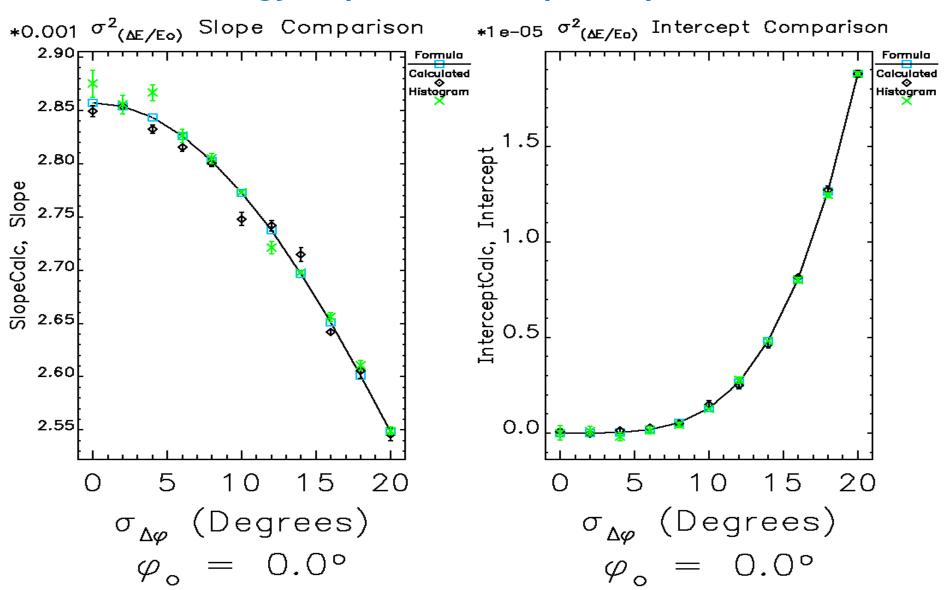 - Fit a line to the relative energy error squared vs relative gradient error squared

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Monte-carlo calculation overview

- Calculate for a linac consisting of:
  - N = 350 cavities (~650 m long)
  - Cavity gradient V_o = 20 MV/m
  - 1 m long cavities
- Generate 100k "linacs" to compute statistics required for the relative energy error
  - Evaluate $<\Delta E>$ and $<\Delta E^2>$ statistics directly
  - Evaluate the statistics from a histogram of the energy error
- Vary gradient and phase error standard deviations over a specified range
- Repeat for two different off-crest phase angles
- Fit a line to the data
- Compare the slope and intercept of the line to formula
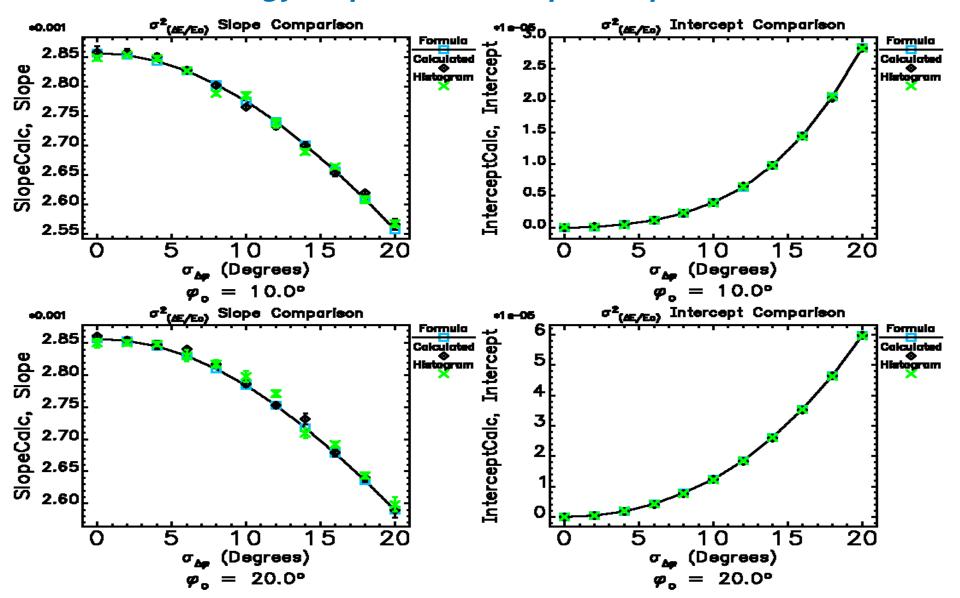- Use computing cluster resources managed by APS (>180 processors in two clusters)

User Beam Energy Slope and Intercept Comparison

User Beam Energy Slope and Intercept Comparison cont.

Effective energy spread

- Definition of the effective energy spread:

$$\sigma^2_{eff} = \sigma^2_{\delta} + \sigma^2_{\left(\frac{\Delta E}{E_f}\right)}$$

- Natural energy spread for user beam (@ 7 GeV) from simulation[1]:
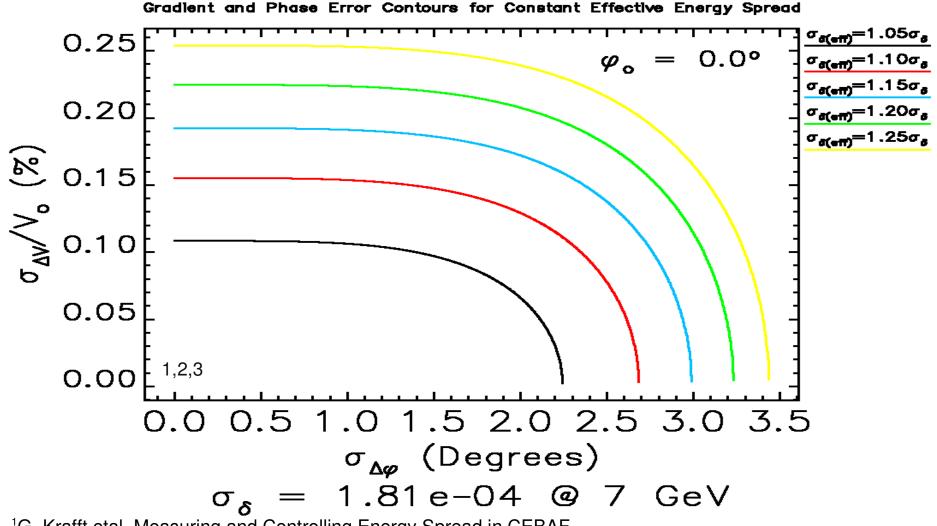
$$\sigma_{\delta} = 0.0181\%$$

- Compute gradient and phase error contours for constant effective energy spread some fraction f above the natural energy spread:

$$\sigma_{eff} = (1 + f)\sigma_{\delta}$$

[1]M. Borland, Comparison of ERL Options and Greenfield ERL.

Talk given to the Machine Advisory Committee for the technical

review of APS Upgrade Options, November 2006.

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Constant effective energy spread contours for user beam on-crest

Gradient and Phase Error Contours for Constant Effective Energy Spread

Legend:
- $\sigma_{\delta(eff)} = 1.05\sigma_\delta$
- $\sigma_{\delta(eff)} = 1.10\sigma_\delta$
- $\sigma_{\delta(eff)} = 1.15\sigma_\delta$
- $\sigma_{\delta(eff)} = 1.20\sigma_\delta$
- $\sigma_{\delta(eff)} = 1.25\sigma_\delta$

$\varphi_o = 0.0°$

Y-axis: $\sigma_{\Delta V}/V_o$ (%)

X-axis: $\sigma_{\Delta\varphi}$ (Degrees)

1,2,3

$\sigma_\delta = 1.81e{-}04 \ @ \ 7 \ GeV$

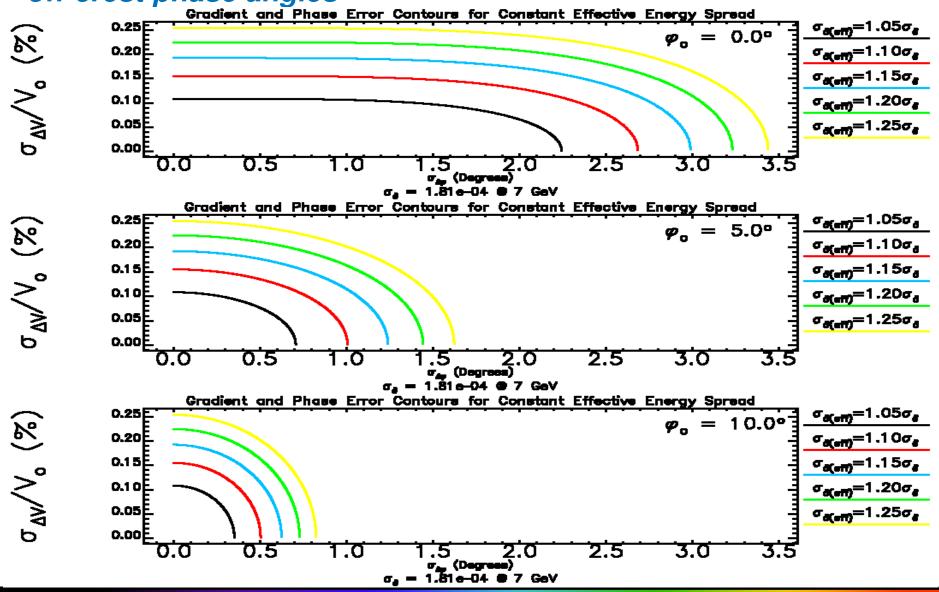[1]G. Krafft etal, Measuring and Controlling Energy Spread in CEBAF

[2] L. Merminga etal, Operation of the CEBAF Linac with High Beam Loading

[3] L. Merminga these proceedings

Comparison of effective energy spread contours for three off-crest phase angles

Energy Recovered Beam Analytic Formula Derivation

- Derivation proceeds similarly to that for user beam:

$$\Delta E = \sum_{n=1}^{N} (V_o + \Delta V_n^{(1)}) \cos(\phi_o^{(1)} + \Delta \phi_n^{(1)})$$

$$- \sum_{n=1}^{N} (V_o + \Delta V_n^{(2)}) \cos(\phi_o^{(2)} + \Delta \phi_n^{(2)}) - E_{sy}$$

$$\sigma^2_{\left(\frac{\Delta E}{E_f}\right)} \equiv \frac{\langle (\Delta E - \langle \Delta E \rangle)^2 \rangle}{E_f^2} = \frac{\langle \Delta E^2 \rangle - \langle \Delta E^2 \rangle}{E_f^2}$$

$$E_f = E_o = 10\,\text{MeV}$$

$$E_{sy} = 15\,\text{MeV}$$

- First and second (energy recovered) beam nominal phases depend on the synchrotron radiation energy loss:

$$\cos \phi_o^{(2)} = \cos \phi_o^{(1)} - \frac{E_{sy}}{N V_o} = 1 - \frac{E_{sy}}{N V_o}$$

Energy Recovered Beam Analytic Formula Derivation cont.

- Formula for the relative energy error has the same form as that for the user beam:

$$\sigma^2_{\left(\frac{\Delta E}{E_f}\right)} = \frac{N}{2}\left(2 + (\cos 2\phi_o^{(1)} + \cos 2\phi_o^{(2)})e^{-2\sigma^2_{\Delta\phi}}\right)\left(\frac{\sigma_{\Delta V}}{V_o}\right)^2$$

$$+ \frac{NV_o^2}{2E_o^2}\left(1 - e^{-\sigma^2_{\Delta\phi}}\right)\left(2 - (\cos 2\phi_o^{(1)} + \cos 2\phi_o^{(2)})e^{-\sigma^2_{\Delta\phi}}\right)$$

- Relative energy error depends linearly on *N* in this case
- Note gradient and phase errors for each pass come from the same gaussian error distribution:

$$\sigma_{\Delta\phi^{(1)}} = \sigma_{\Delta\phi^{(2)}} = \sigma_{\Delta\phi}$$

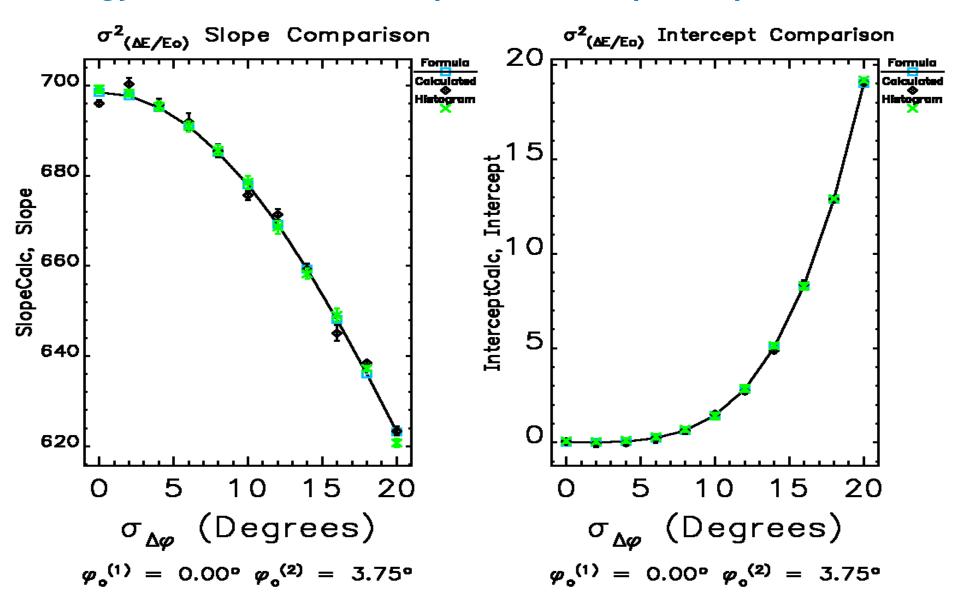$$\sigma_{\Delta V^{(1)}} = \sigma_{\Delta V^{(2)}} = \sigma_{\Delta V}$$

Energy Recovered Beam Analytic Formula Derivation cont.

- Check this formula in a similar fashion using monte-carlo
  - Evaluate the statistics $<\Delta E>$ and $<\Delta E^2>$ directly
  - Evaluate the statistics from a histogram of the energy error
  - Compute slope and intercept for best fit curve as with user beam analysis
  - Use same linac parameters used for first pass beam calculation
  - Use natural energy spread from full, start-to-end elegant tracking simulation[1]
- **Evaluate relative energy error over user beam gradient and phase error range**

[1]M. Borland, Comparison of ERL Options and Greenfield ERL.

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Energy Recovered Beam Slope and Intercept Comparison

Energy Recovered Beam Effective Energy Spread Calculation

$\sigma_{\delta(eff)}$ Contours For the Energy Recovered Beam

$\sigma_{\delta} = 6.8 \% @ 10 \text{ MeV}$

Conclusion

- Gradient and phase error tolerances for user beam modest to keep effective energy spread < 15 % increase above natural energy spread
  - $\sigma_{\Delta\phi}$ < 2 degrees for phase errors
  - $\sigma_{\Delta V}$ < 0.2 % for gradient errors (40 kV out of 20 MV)
- These error tolerances imply ~12 % effective energy spread for the energy recovered beam
  - Desire low losses at the beam dump
  - May impact beam dump design
- How do uncorrectable gradient and phase errors scale to 100 mA?
- Analytic formulas pushed about as far as they can go
- Easily modify the monte-carlo calculations to include more complicated gradient/phase error distributions in the linac
  - Systematic nominal gradient and phase (V_o and ϕ_o) errors
  - Model gradient and phase feedback loop action vs beam current
  - Effect of injector beam
- Useful to compare with tracking simulation using elegant

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